9 Analog Pulse Modulation and Pulse Shaping

Wednesday, September 05, 2012
8:52 AM

Start with $m[n]$ (discrete-time signal
sequence of numbers)

which may come from
1) $m(t) \longrightarrow$ sampling
2) "digital" source
information/message is generated in the form

$$0\,1\,000\,1\,000\,1\,1\,000\ldots$$

$$m[n] = -1 \quad 1 \quad -1 \quad -1 \quad -1 \quad 1$$

9.1 Analog Pulse Modulation

Instead of sending $m(t) \times \cos(2\pi f_c t)$

Let's try to see how we can transmit $m[n]$ directly.

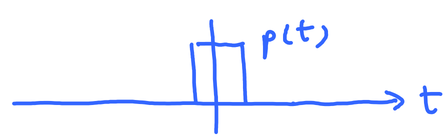
In this section, we will encode this info ($m[n]$) into a sequence of pulses $p(t)$.

We will focus on only one kind of analog pulse modulation:

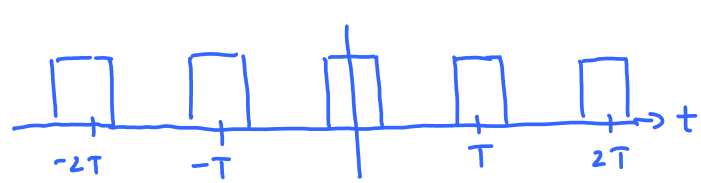
PAM : Pulse Amplitude Modulation

Use the $m[n]$ to "modulate" the amplitude (height) of a pulse train.

Example of a pulse $p(t)$



Unmodulated pulse train: $\sum_{n=-\infty}^{\infty} p(t-nT)$



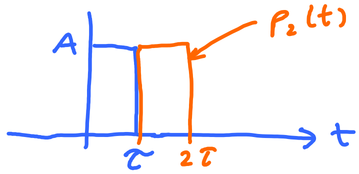
For PAM, the transmitted signal has the form

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} m[n]\,p(t-nT)}$$

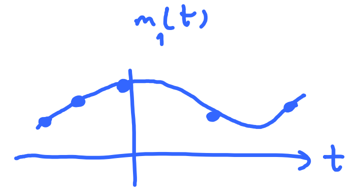
pulse-modulated signal

pulse for user #1

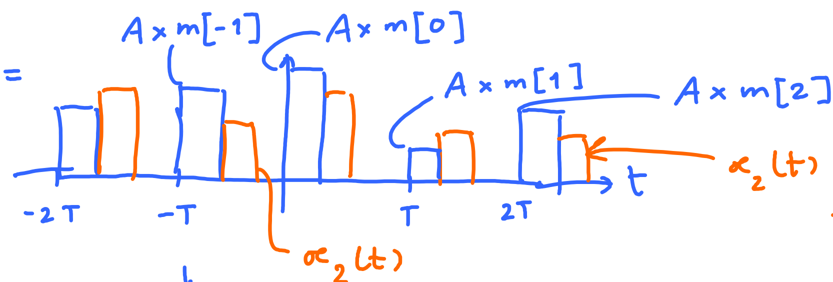
Ex. $p_1(t) =$

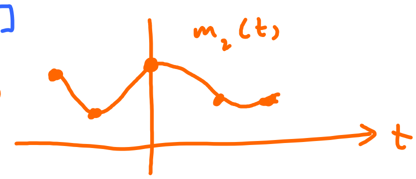


$p_2(t)$ ← pulse for user #2

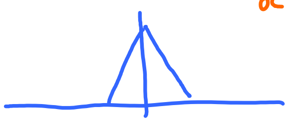


$x_1(t) =$

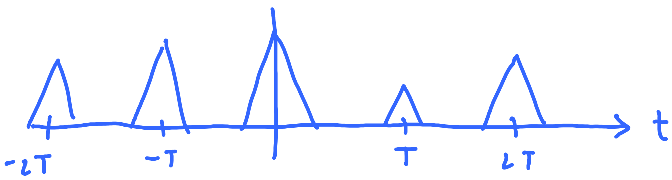




Ex. $p(t) =$



$x(t)$



Q: Why is this useful?

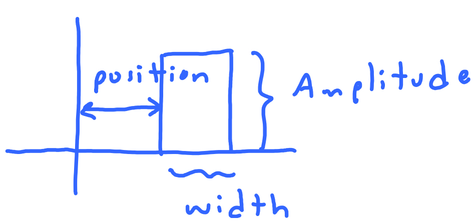
Time sharing → TDM = time division multiplexing

Time sharing → TDMA = time division multiple access

allow transmission of $m_1[n]$ and $m_2[n]$ via $x_1(t)$ and $x_2(t)$, respectively (use different (non-overlapping) parts of time)

Different types of analog pulse modulation

pulse



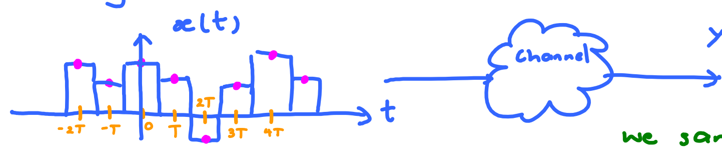
9.2 ISI and Nyquist's Pulse Shaping

↳ inter-symbol interference

Start with sequence of numbers:

$\ldots, m[-3], m[-2], m[-1], m[0], m[1], m[2], \ldots$

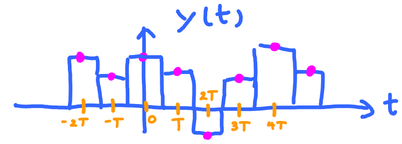
Transmit using PAM



$y(t) = x(t)$ @ receiver,

we sample $y(t)$ @ nT to get $\tilde{m}[n]$

$$x(t) = \sum_n m[n]\, p(t-nT)$$



Note that $\tilde{m}[n] = m[n]$ here

↳ no ISI

Problem: In freq. domain, rectangular pulse becomes sinc

↑ not bandlimited.

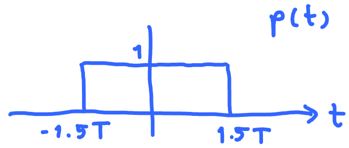
Solution:

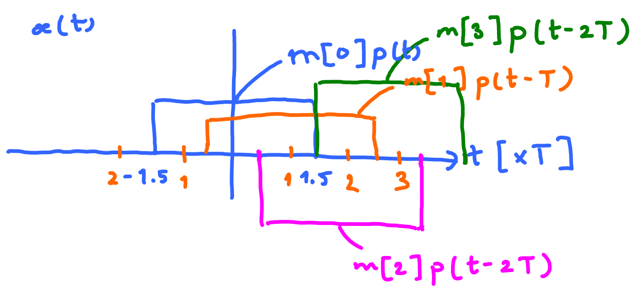
Set a condition (that we need to satisfy):

the pulse $p(t)$ that we want to use has to be bandlimited.

Another Problem: intersymbol interference ..

Let's try





Notice that we have ISI.

$\tilde{m}[0] = m[0] + m[1]$

$\tilde{m}[1] = m[0] + m[1] + m[2]$

$\tilde{m}[2] = m[1] + m[2] + m[3]$

$\tilde{m}[3] = m[2] + m[3]$

we don't want these!

Solution:

$x(t)$

"

Set another condition : when we sample $\ddot{y}(t)$ @ nT,

we want no ISI.

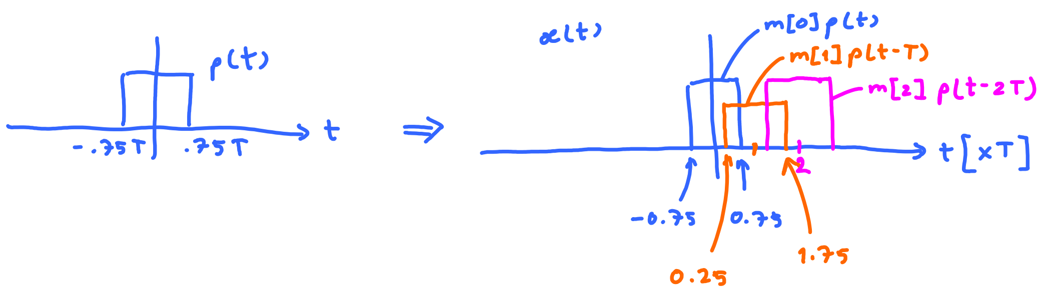
Summary : Two conditions/requirements for $p(t)$

① Band-limited

② No ISI

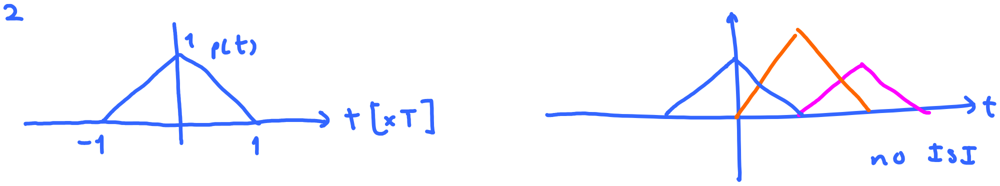
If I use ...

Ex. 1



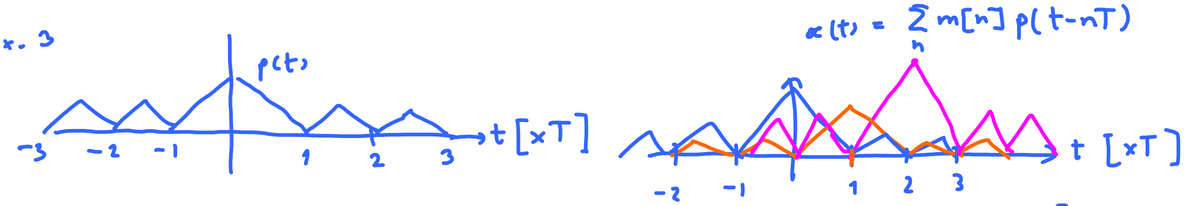
As long as my pulse is time-limited inside (-1,1), we have no ISI.

Ex. 2



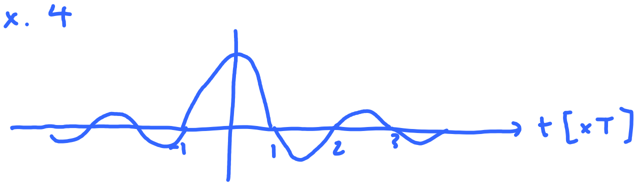
Ex. 3

$x(t) = \sum_n m[n]\, p(t-nT)$



no ISI

Ex. 4



→ no ISI

Advantages - no ISI

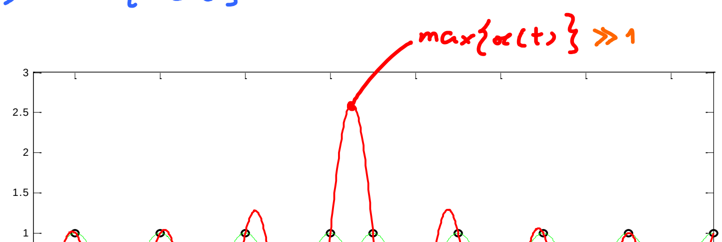
- band limited

Problem - infinite duration

- steep slope at 0-intercept

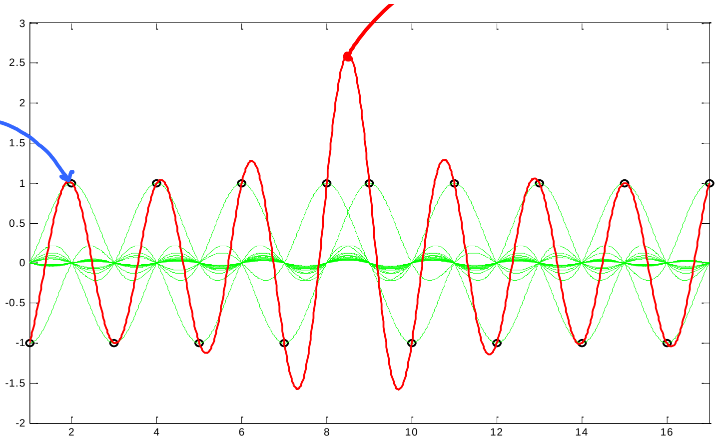
(with cause problem when you have timing / sync. error)

- $\max\{x(t)\} \gg \max\{m[n]\}$



$\max\{m[n]\}$

In this example, $m[n]$ only has two possible values: 1 or -1



Nyquist (first) criterion for zero ISI

There are three of them. However, we will focus on only one of them.

$\hookrightarrow$ the n^{th} sample is not affected by preceding or succeeding sample values.

It is given in freq. domain.

$\Rightarrow$ good because we want to design band-limited pulse(s).

In time domain, we want

$$p[n] = p(t)\Big|_{t=nT} = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

$\leftarrow$ This is the discrete-time version of δ-function.

old technique : DTFT

New technique : $Ш(t)$ $\qquad \sum_n \delta(t-nT)$

$$p_\delta(t) = p(t) \times \underbrace{Ш_T(t)}_{\sum_n \delta(t-nT)} \xrightarrow{\mathcal{F}} P(f) * \frac{1}{T}\sum_k \delta\left(f-\frac{k}{T}\right) = \frac{1}{T}\sum_k P\left(f-\frac{k}{T}\right) = P_\delta(f)$$

$= \delta(t)$ $\qquad\qquad = 1$

Conclusion : If $\sum_k P\left(f-\frac{k}{T}\right) \equiv T$, then no ISI.

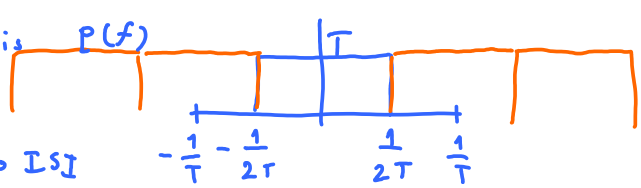
(hold ~~for~~ all f)

Pulses that satisfy this condition are called Nyquist Pulses.

T = symbol duration

stupid .. you only need to check at $-\frac{1}{2T} \le f \le \frac{1}{2T}$

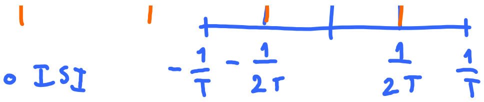
Ex 5 This $P(f)$



No ISI

$\xrightarrow{\mathcal{F}^{-1}}$ $\text{sinc}\left(\frac{1}{T} t\right)$

(Ex. 4)

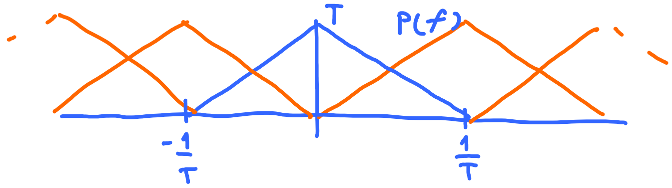


No ISI

$\longrightarrow$ $\mathrm{sinc}(\ldots \frac{\ }{2T} \ldots)$

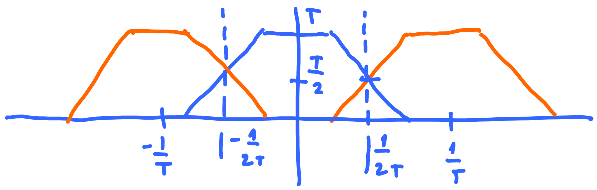
(Ex. 4)

Ex. 6



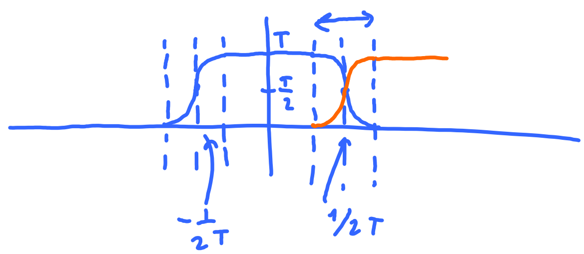
No ISI

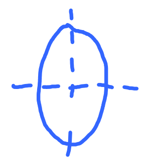
Ex 7



No ISI

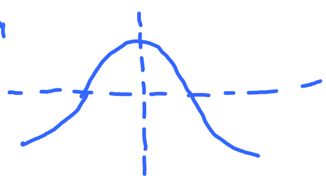
Ex 8.



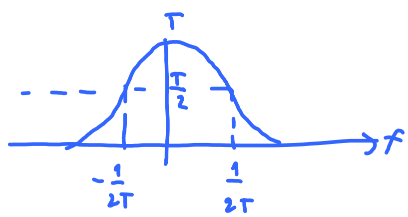


No ISI

Ex. 9



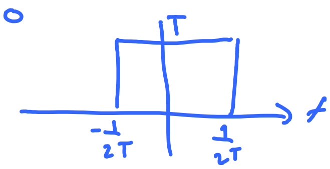
$\alpha = 1$



Raised Cosine Pulse
(RC)

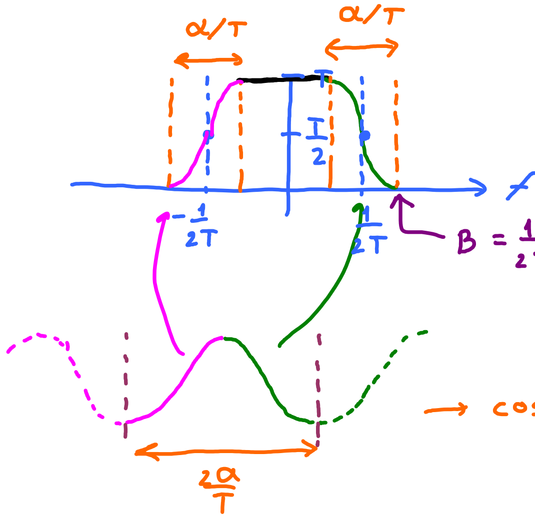
$p_{RC}(t;\alpha)$

$\alpha = 0$



$P_{RC}(f;\alpha)$

For arbitrary α, $\in [0, 1]$

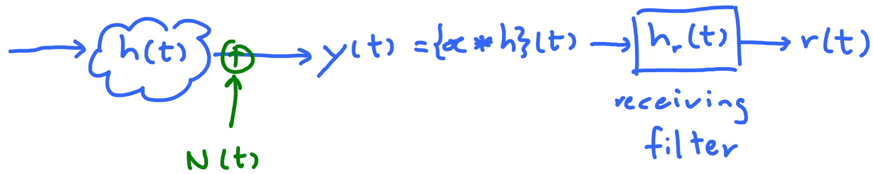


$B = \frac{1}{2T} + \frac{\alpha}{2T} = \frac{1+\alpha}{2T}$

$\longrightarrow \cos\left(\cancel{2}\pi \frac{T}{\cancel{2}\alpha} f\right)$

In practice, ...

$$x(t) = \sum_n m[n]\, p(t-nT)$$



$$r(t) = y(t) * h_r(t) = x(t) * h(t) * h_r(t) = \sum_n m[n]\, p(t-nT) * h(t) * h_r(t)$$

$$= \sum_n m[n]\, \underbrace{\delta(t-nT) * }_{p(t-nT)} \underbrace{p(t) * h(t) * h_r(t)}_{p_{eff}(t)} = \sum_n m[n]\, p_{eff}(t-nT)$$